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THE EFFECT OF PRIMORDIAL TEMPERATURE FLUCTUATIONS ON THE QCD TRANSITION^a

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We analyse a new mechanism for the cosmological QCD first-order phase transition: inhomogeneous nucleation. The primordial temperature fluctuations are larger than the tiny temperature interval, in which bubbles would form in the standard picture of homogeneous nucleation. Thus the bubbles nucleate at cold spots. We find the typical distance between bubble centers to be a few meters. This exceeds the estimates from homogeneous nucleation by two orders of magnitude. The resulting baryon inhomogeneities may affect primordial nucleosynthesis.

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First-order phase transitions normally proceed via nucleation of bubbles of the new phase. When the temperature is spatially uniform and no impurities are present, the mechanism is homogeneous nucleation. We denote by v_{heat} the effective speed by which released latent heat propagates in sufficient amounts to shut down nucleations. The mean distance between nucleation centers, measured immediately after the transition completed, is

$$d_{\text{nuc,hom}} = 2v_{\text{heat}}\Delta t_{\text{nuc}}, \quad (1)$$

where Δt_{nuc} is the duration of the nucleation period^{1,2}.

Surface tension and latent heat are provided by lattice simulations with quenched QCD only, giving the values $\sigma = 0.015T_c^3$, $l = 1.4T_c^4$ ³. Scaling the latent heat for the physical QCD leads us to take $l = 3T_c^4$. Using these values the amount of supercooling is $\Delta_{\text{sc}} = 2.3 \times 10^{-4}$, with large error bars. Due to rapid change of energy density near T_c , the microscopic sound speed in the quark phase has a small value, $3c_s^2(T_c) < 1$ (for references see¹).

In the real Universe the local temperature of the radiation fluid fluctuates. The temperature contrast is denoted by $\Delta \equiv \delta T/\bar{T}$. On subhorizon scales in the radiation dominated epoch, each Fourier coefficient $\Delta(t, k)$ oscillates with constant amplitude $\Delta_T(k)$. Inflation predicts a Gaussian distribution,

$$p(\Delta)d\Delta = \frac{1}{\sqrt{2\pi}\Delta_T^{\text{rms}}} \exp\left(-\frac{1}{2}\frac{\Delta^2}{(\Delta_T^{\text{rms}})^2}\right)d\Delta. \quad (2)$$

We find⁴ for the COBE normalized⁵ rms temperature fluctuation of the radiation fluid (not of cold dark matter) $\Delta_T^{\text{rms}} = 1.0 \times 10^{-4}$ for a primordial Harrison-Zel'dovich spectrum. The change of the equation of state prior to the QCD transition modifies the temperature-energy density relation, $\Delta = c_s^2 \delta\varepsilon/(\varepsilon + p)$. We may neglect the pressure p near the critical temperature since $p \ll \varepsilon_q$ at T_c . On the other hand the drop of the sound speed enhances the amplitude of the density fluctuations proportional to $c_s^{-1/2}$ ⁶. Putting all those effects together and allowing for a tilt in the power spectrum, the COBE normalized rms temperature fluctuation reads

$$\Delta_T^{\text{rms}} \approx 10^{-4} (3c_s^2)^{3/4} \left(\frac{k}{k_0}\right)^{(n-1)/2}. \quad (3)$$

For a Harrison-Zel'dovich spectrum and $3c_s^2 = 0.1$, we find $\Delta_T^{\text{rms}} \approx 2 \times 10^{-5}$.

A small scale cut-off in the spectrum of primordial temperature fluctuations comes from collisional damping by neutrinos⁶. For l_{smooth} , the length over which temperature is constant, we take the value $10^{-4}d_H$ ¹. The temperature fluctuations are frozen with respect to the time scale of nucleations.

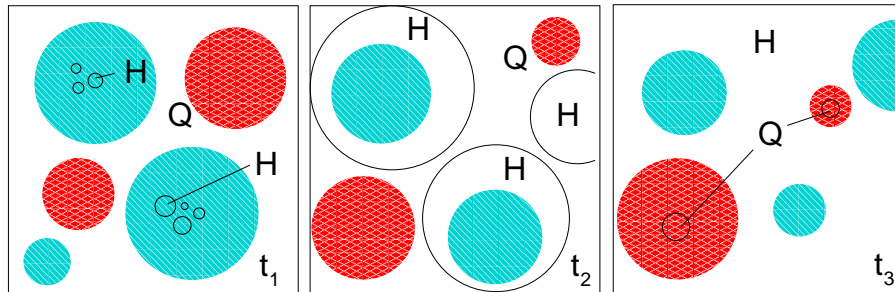


Figure 1: Sketch of a first-order QCD transition in the inhomogeneous Universe. At t_1 the first hadronic bubbles (H) nucleate at the coldest spots (light gray), while most of the Universe remains in the quark phase (Q). At t_2 the bubbles inside the cold spots have merged and have grown to bubbles as large as the temperature fluctuation scale. At t_3 the transition is almost finished. The last quark droplets are found in the hottest spots (dark gray).

Let us now investigate bubble nucleation in a Universe with spatially inhomogeneous temperature distribution. Nucleation effectively takes place while the temperature drops by the tiny amount Δ_{nuc} . To determine the mechanism of nucleation, we compare Δ_{nuc} with the rms temperature fluctuation Δ_T^{rms} :

1. If $\Delta_T^{\text{rms}} < \Delta_{\text{nuc}}$, the probability to nucleate a bubble at a given time is *homogeneous* in space. This is the case of homogeneous nucleation.
2. If $\Delta_T^{\text{rms}} > \Delta_{\text{nuc}}$, the probability to nucleate a bubble at a given time is *inhomogeneous* in space. We call this inhomogeneous nucleation.

The quenched lattice QCD data and a COBE normalized flat spectrum lead to the values $\Delta_{\text{nuc}} \sim 10^{-6}$ and $\Delta_T^{\text{rms}} \sim 10^{-5}$. We conclude that the cosmological QCD transition may proceed via inhomogeneous nucleation. A sketch of inhomogeneous nucleation is shown in Fig. 1. The basic idea is that temperature inhomogeneities determine the location of bubble nucleation. Bubbles nucleate first in the cold regions.

For the fastest fluctuations, with angular frequency c_s/l_{smooth} , we find

$$\frac{dT(t, \mathbf{x})}{dt} = \frac{\bar{T}}{t_H} \left[-3c_s^2 + \mathcal{O} \left(\Delta_T \frac{t_H}{\delta t} \right) \right]. \quad (4)$$

The Hubble expansion is the dominant contribution, as typical values are $3c_s^2 = 0.1$ from quenched lattice QCD and $\Delta_T^{\text{rms}} t_H / \delta t \approx 0.01$. This means that the local temperature does never increase, except by the released latent heat during bubble growth.

To gain some insight in the physics of inhomogeneous nucleation, let us first inspect a simplified case. We have some randomly distributed cold spheres of

diameter l_{smooth} with equal and uniform temperature, which is by the amount $\Delta_T^{\text{rms}} T_c$ smaller than the again uniform temperature in the rest of the Universe. When the temperature in the cold spots has dropped to T_f , homogeneous nucleation takes place in them. Due to the Hubble expansion the rest of the Universe would need the time $\Delta t_{\text{cool}} = t_H \Delta_T^{\text{rms}} / 3c_s^2$ to cool down to T_f . The cold spots have fully been transformed into the hadron phase while the rest of the Universe still is in the quark phase. The latent heat released in a cold spot propagates in all directions, which provides the length scale

$$l_{\text{heat}} \equiv 2v_{\text{heat}} \Delta t_{\text{cool}}. \quad (5)$$

If the typical distance from the boundary of a cold spot to the boundary of a neighboring cold spot is less than l_{heat} , then no hadronic bubbles can nucleate in the intervening space.

The real Universe consists of smooth patches of typical linear size l_{smooth} , their temperatures given by the distribution (2). At time t heat, coming from a cold spot which was transformed into hadron phase at time t' , occupies the volume $V(t, t') = (4\pi/3)[l_{\text{smooth}}/2 + v_{\text{heat}}(t - t')]^3$. The fraction of space that is not reheated by the released latent heat (and not transformed to hadron phase), is given at time t by

$$f(t) \approx 1 - \int_0^t \Gamma_{\text{ihn}}(t') V(t, t') dt', \quad (6)$$

where we neglect overlap and merging of heat fronts. Γ_{ihn} is the volume fraction converted into the new phase, per physical time and volume as a function of the mean temperature $T = \bar{T}(t)$. We find

$$\Gamma_{\text{ihn}} = 3c_s^2 \frac{T_f}{T} \frac{1}{t_H \mathcal{V}_{\text{smooth}}} p(\Delta = \frac{T_f}{T} - 1), \quad (7)$$

where the relevant physical volume is $\mathcal{V}_{\text{smooth}} = (4\pi/3)(l_{\text{smooth}}/2)^3$.

The end of the nucleation period, t_{ihn} , is defined through the condition $f(t_{\text{ihn}}) = 0$. We introduce the variables $N \equiv (1 - T_f/T)/\Delta_T^{\text{rms}}$ and $\mathcal{N} \equiv N(t_{\text{ihn}})$. Putting everything together we determine \mathcal{N} from

$$\frac{l_{\text{heat}}^3}{l_{\text{smooth}}^3} \int_{\mathcal{N}}^{\infty} dN \frac{e^{-\frac{1}{2}N^2}}{\sqrt{2\pi}} \left(\frac{l_{\text{smooth}}}{l_{\text{heat}}} + N - \mathcal{N} \right)^3 = 1. \quad (8)$$

For $l_{\text{heat}}/l_{\text{smooth}} = 1, 2, 5, 10$ we find $\mathcal{N} \approx 0.8, 1.4, 2.1, 2.6$, respectively.

The effective nucleation distance in inhomogeneous nucleation is defined from the number density of those cold spots that acted as nucleation centers,

$d_{\text{nuc,ihn}} \equiv n^{-1/3}$. We find

$$d_{\text{nuc,ihn}} \approx \left[\int_0^{t_{\text{ihn}}} \Gamma_{\text{ihn}}(t) dt \right]^{-1/3} \quad (9)$$

$$= \left[\frac{3}{\pi} (1 - \text{erf}(\mathcal{N}/\sqrt{2})) \right]^{-1/3} l_{\text{smooth}}. \quad (10)$$

With the above values $l_{\text{heat}}/l_{\text{smooth}} = 1, 2, 5, 10$ we get $d_{\text{nuc,ihn}} = 1.4, 1.8, 3.0, 4.8 \times l_{\text{smooth}}$, where $l_{\text{smooth}} \approx 1$ m.

For a COBE normalized spectrum without any tilt and with a tilt of $n - 1 = 0.2$, together with $3c_s^2 = 0.1$ and $v_{\text{heat}} = 0.1$, we find the estimate $l_{\text{heat}}/l_{\text{smooth}} \approx 0.4$ and 9, correspondingly. Notice that the values of v_{heat} and $3c_s^2$ are in principle unknown. Anyway, we can conclude that the case $l_{\text{heat}} > l_{\text{smooth}}$ is a realistic possibility.

With $2v_{\text{heat}}(3c_s^2)^{-1/4}(10^{-4}d_{\text{H}}/l_{\text{smooth}}) < 1$ and without positive tilt we are in the region $l_{\text{heat}} < l_{\text{smooth}}$, where the geometry is more complicated and the above quantitative analysis does not apply. In this situation nucleations take place in the most common cold spots ($\mathcal{N} \sim 1$), which are very close to each other. We expect a structure of interconnected baryon-depleted and baryon-enriched layers with typical surface l_{smooth}^2 and thickness $l_{\text{def}} \equiv v_{\text{def}}\Delta t_{\text{cool}}$.

We emphasize that inhomogeneous and heterogeneous nucleation⁷ are genuinely different mechanisms, although they give the same typical scale of a few meters by chance. If latent heat and surface tension of QCD would turn out to reduce Δ_{sc} to, e.g., 10^{-6} , instead of 10^{-4} , the maximal heterogeneous nucleation distance would fall to the centimeter scale, whereas on the distance in inhomogeneous nucleation this would have no effect.

We have shown that inhomogeneous nucleation during the QCD transition can give rise to an inhomogeneity scale exceeding the proton diffusion scale (2 m at 150 MeV). The resulting baryon inhomogeneities could provide inhomogeneous initial conditions for nucleosynthesis. Observable deviations from the element abundances predicted by homogeneous nucleosynthesis seem to be possible in that case⁸.

In conclusion, we found that inhomogeneous nucleation leads to nucleation distances that exceed by two orders of magnitude estimates based on homogeneous nucleation. We point out that this new effect appears for the (today) most probable range of cosmological and QCD parameters.

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